**Slide 2 - Mathematical Formalism of open quantum systems**

* Open quantum system - quantum system in contact with environmental degrees of freedom
* System is described by Hamiltonian H\_S, environment by H\_E, and the interaction between them by H\_I
* Density operator encapsulates all info
* Evolution in time given by von-Neumann
* Partition the hamiltonian into soluble system part and complex interaction part which is treated as a perturbation
* Transform to interaction picture via unitary transform to simplify things
* Actually only need work with the reduced density operator

**Slide 3 - Master equations in Lindblad form**

* Transform von-Neumann equation to interaction picture
* Solve recursively and use several approximations, namely: no system environment correlations at t=0 which is extended to all time by Born approx, thermal environment, Markov approximation so equation is memoryless
* Choose a specific form of Hamiltonian, transformed to the interaction picture.
* Expand commutators, sub in decomposed Hamiltonian to get interaction picture master equation
* We have introduced environment correlation functions defined as shown, in the case of a stationary environment they are simplified

**Slide 4 - Spin-boson model**

* Interested in 2-level systems which are described by the spin-boson hamiltonian
* Substitute into the interaction picture master equation and transform back to the Schroedinger picture to get the secular master equation
* The lamb shift term is often small and so ignored
* The P operators are defined as shown so P\_0 is a phase change while P\_eta changes the state
* For reasons we will see in a second, we choose to work in the eigenbasis of the system hamiltonian, whose basis states are +/-
* The rates at which these different processes occur are shown. We use a simple spectral density of ohmic form in which case they are simplified.
* We have introduced this eta term defined as shown,

**Slide 5 - Quantum trajectories method**

* Numerical method to unravel master equations
* Has advantages in reducing the dimensionality of the problem and provides nice physical insight
* Can be parallelised since trajectories are independent which we make use of for expensive calculations
* Master equations can be written in what is called Lindblad form.
* Subsequently we can define an effective Hamiltonian
* In the case of the spin-boson model, the effective Hamiltonian is shown.
* This effective Hamiltonian is then used in the quantum trajectories algorithm.
* We covered the details of the algorithm and its equivalence to solving the master equation last semester.
* The basic principle is that you calculate a candidate state by propagating a state with the effective hamiltonian. The norm of this candidate state determines whether a quantum jump occurs , in which case we act with the appropriate jump operator, or if not the final state is proportional to the candidate state.
* The energy basis is intuitive as the system will remain in its eigenstate unless a jump occurs.

**Slide 6 - Quantum adiabatic theorem**

* Consider no environment and a simple system hamiltonian without tunnelling, i.e delta = 0.
* Its eigenstates are up and down, known as the diabatic states.
* The diabatic states have energies linear in epsilon (dashed lines)
* Add in a tunnelling term and the eigen-energies now change as shown.
* The basis states are now +/-, known as adiabatic states, with energies that have hyperbolic form (solid lines).
* The graph shows the energies of these 4 states in two different cases
* The left case is where delta = 0 and the diabatic and adiabatic energies overlap with a degeneracy at epsilon = 0.
* When delta =/= 0 the adiabatic state energies are separated by delta at epsilon = 0 and demonstrate a phenomena known as an avoided crossing.
* Adiabatic theorem states that a physical system prepared in its instantaneous eigenstate will remain in that state if the perturbation is acting slowly enough and there is a gap between the eigenvalue and the rest of the Hamiltonians spectrum.
* So we fix delta at a small constant value and ramp epsilon from a large negative to large positive value.
* If we start in - (close to up) and ramp adiabatically, we follow the solid red line and end up in - but the diabatic states have flipped so we close to down.
* If we ramp diabatically we follow the dashed red line and our adibatic basis flips so we end up in + but the diabatic basis does not so we are still close to up.

**Slide 7 - Quantum adiabatic theorem - No environment**

* Can demonstrate the above idea with the quantum trajectories method by plotting the populations of various states as we ramp epsilon at different speeds.
* Left hand plot shows the adiabatic state case i.e +/-
* In the diabatic limit we see a transition but no transition in the adiabatic case
* Right hand plot shows the diabatic state case i.e. up/down
* In the diabatic limit we observe so transition but we see on in the adiabatic limit

**Slide 8 - Quantum adiabatic theorem - Environment**

* The environment complicates things
* We consider the diabatic states which is relevant for our upcoming discussion
* We see that increasing the temperature or the coupling strength the environment has more of an effect, inducing more quantum jumps, and ultimately pulling us away from the case of no environment

**Slide 9 - The Landau-Zener formula**

* In the case of no environment with a linearly changing perturbation where the time-dependent component does not couple our states there exists an analytic solution for comparison.
* These are the scenarios we have considered when ramping epsilon and fixing delta
* The formulas for a diabatic and adiabatic transition are shown, the key term is Gamma which depends on the speed at which we ramp epsilon.
* The graphs show these formulas plotted alongside the state populations for both the diabatic basis (left) and adiabatic basis (right).
* The x-axis is logarithmic in ramp speed so the far left is adiabatic and the far right is diabatic
* We start in - (close to up) thus for the diabatic basis we see a transition in the adiabatic limit and no transition in the diabatic limit, with the opposite occurring for the adiabatic basis which is consistent with previous results.

**Slide 10 - Heat and work**

* Now we turn to considering thermodynamic properties
* 2-level systems have many applications but quantum computing is a well known one
* It utilises quantum gates, the most simple being an X-gate or quantum NOT gate which simply flips the state so we can draw parallels between this and the transition discussed.
* Might be interesting to know how work and heat costs change as we change the regime
* In the case of no environment the work done in the adiabatic limit is simply zero as the energy state does not change but it is finite in the diabatic limit
* When an environment is present, the total energy change over a time step is shown.
* We adopt the convention of defining work by changing the hamiltonian and using the final state, while heat is defined with a constant hamiltonian but the state can change if a jump occurs.
* When a jump occurs both the heat and the work are fintie.
* If no jump occurs we have a similar case to before, with no heat and all the energy is in the work

**Slide 11 - Heat and work Part 2**

* The ideas of the previous slides are illustrated here.
* The left hand plots show the case with no environment
* The left sub plot shows the amount of work done on the system and the right sub plot shows the spin state populations
* We see that in the adiabatic limit we do no work and flip the spin perfectly so this is the optimal regime
* The right hand plot shows the case with an environment.
* In the same adiabatic limit we now have to do a finite amount of work as there is some heat dissipated and we do not flip the spin as well.
* Despite this, it would still be the most optimal regime

**Slide 12 - Entropy Balance**

* An environment means we do finite work while not flipping the spin perfectly due to the fact that some of the work we do in ramping epsilon is not returned due to being dissipated as heat.
* The process is thus irreversible so it must have an associated entropy production.
* The entropy balance equation for the total production rate is shown.
* It has both a von-Neumann entropy associated with the purity of the state and an entropy flux which defines the entropy that flows from the system to the environment, i.e. it is proportional to the number of quantum jumps.
* The plot shows the magnitude of these components in different regimes
* In the diabatic limit the environment does not have time to act so the entropy production is zero.
* As we slow the ramp the environment begins to induce jumps and the final state is not pure so both quantities are finite.
* In an even slower regime we see the quantities decay to zero, because the potential increase in the total amount of entropy produced from the environment having longer to act does not offset the slowness of the regime.

**Slide 13 - Extensions and Conclusions**

Thank you for coming to our TED talk